

# Afterglows of Mildly Relativistic Supernovae: Baryon Loaded Blastwaves

Sayan Chakraborti and Alak Ray

*Department of Astronomy and Astrophysics, Tata Institute of Fundamental Research,  
1 Homi Bhabha Road, Mumbai 400 005, India*

**Abstract.** Relativistic supernovae have been discovered until recently only through their association with long duration Gamma Ray Bursts (GRB). As the ejecta mass is negligible in comparison to the swept up mass, the blastwaves of such explosions are well described by the Blandford-McKee (in the ultra relativistic regime) and Sedov-Taylor (in the non-relativistic regime) solutions during their afterglows. However, the recent discovery of the relativistic supernova SN 2009bb, without a detected GRB, has indicated the possibility of highly baryon loaded mildly relativistic outflows which remains in nearly free expansion phase during the radio afterglow. In this work, we consider the dynamics and emission from a massive, relativistic shell, launched by a Central Engine Driven EXplosion (CEDEX), decelerating adiabatically due to its collision with the pre-explosion circumstellar wind profile of the progenitor. We show that this model explains the observed radio evolution of the prototypical SN 2009bb and demonstrate that SN 2009bb had a highly baryon loaded, mildly relativistic outflow.

**Keywords:** Blast Waves; Supernovae; Relativistic Fluid flow

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## INTRODUCTION

Gamma-ray Bursts (GRBs) have long been recognized to require ultra relativistic bulk motion of matter particles (see Piran [1, 2] for reviews). GRB afterglows are generated from the emission by relativistic shocks that result from slowing down of a relativistic shell by the the medium surrounding the progenitor star that exploded. In core collapse supernovae similar interaction of stellar material (ejecta) from an exploding star with the circumstellar matter (CSM) results in non-relativistic shocks.

Fluid dynamics of ultra-relativistic spherical blast waves mediated by strong shocks has been treated by Blandford and McKee [3, 4]. They found a similarity solution of an explosion of a fixed amount of energy in a uniform medium. On the other hand, Chevalier [5], Nadezhin [6] described the initial nearly free expansion of a non-relativistic supernova blast wave, interacting with the surrounding circumstellar medium. Once the blast wave sweeps up more CSM material than its own rest mass, the self-similar solutions of non-relativistic blast waves are described in the Newtonian regime by the Sedov [7] von Neumann [8] Taylor [9] solution.

In this presentation we provide an analytic solution (see ApJ [10] for details) of the standard model of relativistic hydrodynamics [see e.g. 1, 11] for an adiabatic blastwave. Here, the exploding shell decelerates due to inelastic collision with an external medium. The solution provided here is for an arbitrary Lorentz factor of the expanding supernova shell. This solution which can handle a trans-relativistic outflow is motivated by the discovery of SN 2009bb, a type Ibc supernova without a detected GRB which shows clear evidence of a mildly relativistic outflow powered by a central engine [12]. SN 2009bb-like objects (Central Engine Driven Explosions, hereafter CEDEX [10]) differ in another significant way from classical GRBs: our work shows that they are highly baryon loaded explosions with non-negligible ejecta masses. The new analytic blastwave solution here therefore generalizes the Blandford and McKee [3] result, in particular their impulsive, adiabatic blast wave in a wind-like  $\rho \propto r^{-2}$  CSM.

## RELATIVISTIC BLASTWAVE SOLUTION

We use the simple collisional model described by Piran [1], Chiang and Dermer [11] where the relativistic ejecta forms a shell which decelerates through infinitesimal inelastic collisions with the circumstellar wind profile. The initial conditions are characterized by the rest frame mass  $M_0$  of the shell launched by a CEDEX and its initial Lorentz

factor  $\gamma_0$ .

The shell slows down by collision with the circumstellar matter. The swept up circumstellar matter is given by  $m(R)$ . Conservation of energy and momentum give us [see 11, 1],

$$\frac{d\gamma}{\gamma^2 - 1} = -\frac{dm}{M} \quad \text{and} \quad dE = c^2(\gamma - 1)dm, \quad (1)$$

respectively, where  $dE$  is the kinetic energy converted into thermal energy, that is the energy in random motions as opposed to bulk flow, by the infinitesimal collision.

For a circumstellar medium set up by a steady wind, where we expect a profile with  $\rho \propto r^{-2}$ , we have  $m(R) = AR$  where  $A$  is the mass swept up by a sphere per unit radial distance.  $A \equiv \dot{M}/v_{wind}$  can be set up by a steady mass loss rate of  $\dot{M}$  with a velocity of  $v_{wind}$  from the pre-explosion CEDEX progenitor, possibly a Wolf Rayet star. Integrating the right hand side and solving for  $\gamma > 1$  this equation we have

$$\gamma = \frac{\gamma_0 M_0 + AR}{\sqrt{M_0^2 + 2A\gamma_0 R M_0 + A^2 R^2}}, \quad (2)$$

which gives the evolution of  $\gamma$  as a function of  $R$ . The amount of kinetic energy converted into thermal energy when the shell reaches a particular  $R$  can be obtained by integrating Equation (1) after substituting for  $\gamma$  from Equation (2) and  $dm = AdR$ , to get

$$E = c^2 \left( -M_0 - AR + \sqrt{M_0^2 + 2A\gamma_0 R M_0 + A^2 R^2} \right). \quad (3)$$

The evolution of  $R$  and  $\gamma$  can be compared with observations once we have the time in the observer's frame that corresponds to the computed  $R$  and  $\gamma$ . For emission along the line of sight from a blastwave with a constant  $\gamma$  the commonly used expression [13] is  $t_{obs} = R/(2\gamma^2 c)$ . However, Sari [14] has pointed out that for a decelerating ultra-relativistic blastwave the correct  $t_{obs}$  is given by the differential equation  $dt_{obs} = dR/(2\gamma^2 c)$ . We substitute  $\gamma$  from Equation (2) and integrate both sides to get the exact expression

$$t_{obs} = \frac{R(M_0 + A\gamma_0 R)}{2c\gamma_0(\gamma_0 M_0 + AR)}. \quad (4)$$

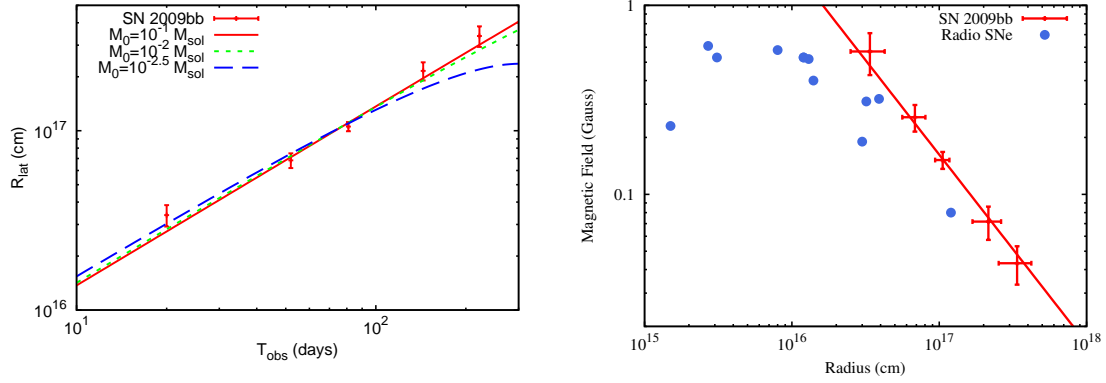
Note that, this reduces to the [13] expression only in the case of nearly free expansion and deviates as the shell decelerates. In the rest of the work we use  $t$  to indicate the time  $t_{obs}$  in the observer's frame. Inverting this equation and choosing the physically relevant *growing branch*, gives us the analytical time evolution of the line of sight blastwave radius, as

$$R = \frac{1}{2A\gamma_0} \times \left( -M_0 + 2Ac\gamma_0 t + \sqrt{8AcM_0 t \gamma_0^3 + (M_0 - 2Ac\gamma_0 t)^2} \right), \quad (5)$$

in the ultra-relativistic regime. This can now be substituted into Equations (2 and 3) to get the time evolution of the Lorentz factor  $\gamma$  and the thermal energy  $E$  [10]. This completes the solution for the blastwave time evolution, parametrized by the values for  $\gamma_0$ ,  $M_0$  and  $A$ .

## BLASTWAVE ENERGETICS

Chakraborti and Ray [10] use the blastwave solution developed in the previous section to predict the radio evolution of a CEDEX with a relativistic blastwave slowing down due to circumstellar interaction. For the prototypical SN 2009bb, the blastwave was only mildly relativistic at the time of the observed radio afterglow. In the absence of a significant relativistic beaming, the observer would receive emission from the entire shell of apparent lateral extent  $R_{lat}$  at a time  $t_{obs}$  given by  $dt_{obs} = dR_{lat}/(\beta\gamma c)$ . Integrating term by term gives us the time evolution of the lateral radius as  $R_{lat} = c\sqrt{\gamma_0^2 - 1}t - O(t^2)$ . The thermal energy available when the shell has moved out to a radius  $R$  is given exactly by Equation (3), however it is again convenient to look at its Taylor expansion,  $E = Ac^2(\gamma_0 - 1)R - O(R^2)$ .



**FIGURE 1. A:** Evolution of the blast wave radius  $R_{lat}$ , determined from SSA fit to observed radio spectrum, as a function of the observer frame time  $t_{obs}$ . The evolution is consistent with nearly free expansion and observations require  $M_0 \gtrsim 10^{-2.5} M_\odot$ . **B:** Magnetic field as a function of blast wave radius, as determined from SSA fits. Blue dots represent size and magnetic field of radio supernovae from Chevalier [15] at peak radio luminosity. Red crosses (with  $3\sigma$  error-bars) give the size and magnetic field of SN 2009bb at different epochs, from spectral SSA fits. Red line gives the best  $B \propto R^{-1}$  (Equation 7) fit.

We consider a electron distribution with an energy spectrum  $N_0 E^{-p} dE$ , which we assume for simplicity to be extending from  $\gamma_m m_e c^2$  to infinity, filling a fraction  $f$  of the spherical volume of radius  $R$ . If a fraction  $\epsilon_e \equiv E_e/E$  of the available thermal energy goes into accelerating these electrons, then for the leading order expansion of  $E$  in  $R$  the normalization of the electron distribution is given by

$$N_0 \simeq \frac{3Ac^2 \epsilon_e (\gamma_0 - 1) (\gamma_m m_e c^2)^2}{2f\pi R^2}, \quad (6)$$

for  $p = 3$ , as inferred from the optically thin radio spectrum of SN 2009bb [12].

We consider a magnetic field of characteristic strength  $B$  filling the same volume. If a fraction  $\epsilon_B \equiv E_B/E$  goes into the magnetic energy density, then the characteristic magnetic field is given by

$$B \simeq \frac{c}{R} \sqrt{\frac{6A\epsilon_B(\gamma_0 - 1)}{f}} \quad (7)$$

This explains the observed  $B \propto R^{-1}$  behavior (Figure 1) seen in SN 2009bb. These observations strengthens the case for a nearly constant  $\epsilon_B$ .

Note that the highest energy to which a cosmic ray proton can be accelerated is determined by the  $BR$  product [16, 17]. We have argued elsewhere [18] that the mildly relativistic CEDEX are ideal for accelerating nuclei to the highest energies to explain the post GZK cosmic rays.

## EXTRACTING BLAST WAVE PARAMETERS FROM RADIO OBSERVATIONS

The inverse problem is that of determining the initial bulk Lorentz factor specified by  $\gamma_0$ , progenitor mass loss rate given by  $A$  or  $\dot{M}$  and the initial ejecta mass  $M_0$ , from the radio observations. The bulk Lorentz factor may be determined from the radio observations using to get the simplified expression for  $\gamma_0$  as [10],

$$\gamma_0^2 \simeq 1 + 0.225 \times \left(\frac{\epsilon_B}{\epsilon_e}\right)^{2/19} \left(\frac{f}{0.5}\right)^{-2/19} \left(\frac{t_{obs}}{20 \text{ days}}\right)^{-2} \left(\frac{v_p}{10 \text{ GHz}}\right)^{-2} \left(\frac{F_{vp}}{20 \text{ mJy}}\right)^{18/19} \left(\frac{D}{40 \text{ Mpc}}\right)^{36/19}. \quad (8)$$

The result is insensitive to the equipartition parameter  $\alpha \equiv \epsilon_e/\epsilon_B$  and filling fraction  $f$ . This may be used to reliably determine the initial bulk Lorentz factor of a radio detected CEDEX in the mildly relativistic, nearly free expansion phase (like SN 2009bb).

A simplified expression for  $A \equiv \dot{M}/v_{wind}$ , the circumstellar density profile, set up by the mass loss from the progenitor has been derived by Chakraborti and Ray [10]. This gives us the approximate expression for the mass

loss rate as

$$\dot{M} \simeq 3.0 \times 10^{-6} \left( \frac{\epsilon_B}{0.33} \right)^{-11/19} \left( \frac{\epsilon_e}{0.33} \right)^{-8/19} \left( \frac{f}{0.5} \right)^{11/19} \left( \frac{v_{wind}}{10^3 \text{ kms}^{-1}} \right)^1 \times \left( \frac{t_{obs}}{20 \text{ days}} \right)^2 \left( \frac{v_p}{10 \text{ GHz}} \right)^2 \left( \frac{F_{vp}}{20 \text{ mJy}} \right)^{-4/19} \left( \frac{D}{40 \text{ Mpc}} \right)^{-8/19} M_{\odot} \text{yr}^{-1}. \quad (9)$$

This approximate expression indicates the dependence of the inferred mass loss rate on the observational parameters, and makes an error of only  $\lesssim 10\%$  in case of SN 2009bb, when compared to our exact expression. Note that, this expression has similar scaling relations as Equation (23) of Chevalier and Fransson [19]. Hence, the mass loss rate of SN 2009bb as determined using that equation by Soderberg et al. [12] remains approximately correct.

The initial ejecta rest mass  $M_0$  cannot be estimated from radio observations in the nearly free expansion phase. It can only be determined when the CEDEX ejecta slows down sufficiently due to interaction with the circumstellar matter (Figure 1). Thereafter, the initial ejecta mass can be obtained using the timescale of slowdown  $t_{dec}$  and the already determined  $A$  and  $\gamma_0$ . Nearly free expansion for a particular period of time, can only put lower limits on the ejecta mass, as shown in this work.

## DISCUSSIONS

Chakraborti and Ray [10] provide a solution of the relativistic hydrodynamics equations which is uniquely tuned to the CEDEX class of objects like SN 2009bb. Because of the non-negligible initial ejecta mass of such a CEDEX, objects like this would persist in the free expansion phase for quite a long time into their afterglow. Sweeping up a mass equal to that of the original ejecta would take considerable time, unless the mass loss scale in its progenitor was very intense, i.e. it had a large  $A$ . We refer the reader to Chakraborti and Ray [10] for a comparison of the CEDEX solution to the intermediate Blandford-McKee like solution and a final Snowplough phase. We also invert the dependence of the observed parameters on  $\gamma_0$  and  $\dot{M}$  in terms of the peak frequency, peak time and peak fluxes to interpret the parameters of the explosion.

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